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Two-Band Antenna-Matching Networks

How They Work and How To Design Them

BY JOHN G. MARSHALL,* WØARL

> **• Continuing correspondence about the two-band matching network described in *QST* a few years ago has prompted WØARL to delve into the subject in greater detail. This article is the result. It covers all practical cases of antennas working on two harmonically-related amateur bands, with straightforward design formulas requiring none of the cut-and-try that was necessary in the original system.**

There has been much interest in the two-band antenna-matching network which was described in *QST* about four years ago.[1] One striking example is the description of a rotary beam which operates as a four-element on 14 Mc. and as an eight-element system on 28 Mc.[2] Such operation is accomplished through the use of one of those two-band antenna networks in conjunction with another type of network in the center of the parasitic elements, the detailed description of which also was given previously in *QST*.[3]

The network described in the early article[1] is suitable when the transmission line's characteristic impedance, Z_0, lies between the two values of driving-point impedance (d.p.i.) and the larger d.p.i. is at the higher operating frequency. It is the purpose of this article to describe two-band antenna networks that will match any combination of Z_0 and d.p.i.s likely to be found in present-day antennas, and also to include more information and more convenient formulas for the previously described case of $Z_1 < Z_0 < Z_2$ (Z_1 being the d.p.i. at the lower operating frequency, f_1, and Z_2 being the d.p.i. at the higher operating frequency, f_2). These networks are not restricted to a second harmonic relationship, but may be used on any two frequencies where the d.p.i. is purely resistive. This permits the use of the same antenna, simultaneously matched to the transmission line, on its fundamental and *any* of its harmonic frequencies, or on any two of its harmonic frequencies where f_2 is an integral multiple of f_1. This includes the 21-Mc. band in connection with either the 3.5- or the 7-Mc. band.

Before entering into the design and operation of such networks, it might be well to review briefly the characteristics of series and parallel reactive circuits that are important to these two-band networks.

Purely Reactive Circuits

Fig. 1A shows a theoretically perfect series circuit of inductive and capacitive reactance. It is generally known that the net reactance, $X = X_L - X_C$; and, when $X_L = X_C$, the circuit is series resonant, and X equals zero.

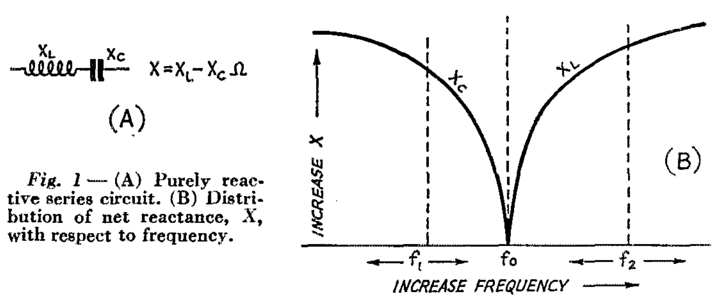

Fig. 1 — (A) Purely reactive series circuit. (B) Distribution of net reactance, X, with respect to frequency.

Fig. 1B shows the distribution of X at frequencies below and above the resonant frequency, f_0. At frequencies below f_0, X is capacitive; at frequencies above f_0, X is inductive. The greater the ohmic values of X_L and X_C, and the greater the deviation from f_0, the greater is the magnitude of X.

When this simple series circuit is viewed from two different frequencies, f_1 and f_2, certain interesting relations exist between the net reactance at f_1 (X_{f1}) and the net reactance at f_2 (X_{f2}). Those which are important to these two-band networks are:

1) When f_0 is greater than f_1 and f_2, both X_{f1} and X_{f2} are capacitive, but X_{f1} is always greater than KX_{f2}, where K is the frequency ratio, f_2/f_1.

2) When f_0 is less than f_1 and f_2, both X_{f1} and X_{f2} are inductive, but X_{f2} is always greater than KX_{f1}.

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[1] Marshall, "Matching the Antenna for Two-Band Operation, *QST*, Sept., 1945.

[2] Pichitino, "A New Principle in Two-Band Rotary-Beam Design," *QST*, Oct., 1948.

[3] Espy, "Resonant Circuits in Antenna Systems," *QST*, Sept., 1943.

Terminology

f_0 — Resonant frequency.
f_1 — Lower operating frequency.
f_2 — Higher operating frequency.
K — Frequency ratio $= f_2/f_1$.
Z_0 — Characteristic impedance of transmission line.
d.p.i. — Driving-point impedance (general).
Z_1 — d.p.i. at f_1.
Z_2 — d.p.i. at f_2.
L_P — Parallel inductor.
X_{LP} — Reactance of L_P at f_1.
C_P — Parallel capacitor.
X_{CP} — Reactance of C_P at f_1.
L_S — Series inductor.
X_{LS} — Reactance of L_S at f_1.
C_S — Series capacitor.
X_{CS} — Reactance of C_S at f_1.
X_{P1} — Net parallel reactance at f_1.
X_{P2} — Net parallel reactance at f_2.
X_{B1} — Net balancing reactance at f_1.
X_{B2} — Net balancing reactance at f_2.

3) When f_0 is between f_1 and f_2, X_{f1} is capacitive, and X_{f2} is inductive.

4) When $f_0 = f_1$, X_{f1} is zero, and X_{f2} is inductive.

5) When $f_0 = f_2$, X_{f2} is zero, and X_{f1} is capacitive.

Formulas for determining the amounts of X_L and X_C required for obtaining desired values of X_{f1} and X_{f2} are easily derived as follows:

At f_1, the net reactance is

$$X_{f1} = X_L - X_C \text{ ohms.}$$

where X_L and X_C are the reactance values at f_1. Since X_L varies directly with frequency and X_C varies inversely with frequency, the net reactance at f_2 is

$$X_{f2} = KX_L - \frac{X_C}{K} \text{ ohms.}$$

Simultaneous solution of these two equations gives the reactance of X_L and X_C at the f_1 frequency as

$$X_L = \frac{KX_{f2} - X_{f1}}{K^2 - 1} \text{ ohms} \qquad (1)$$

and

$$X_C = \frac{K(X_{f2} - KX_{f1})}{K^2 - 1} \text{ ohms.} \qquad (2)$$

The appropriate signs for X_{f1} and X_{f2} should be entered in these expressions. However, since it has been taken into account that X_L is positive and X_C is negative in deriving formulas (1) and (2), a resulting negative answer for either X_L or X_C indicates that an unworkable combination X_{f1} and X_{f2} exists. If one of the above five conditions is satisfied, the answers to formulas (1) and (2) will be positive.

Fig. 2A shows a theoretically perfect parallel circuit of inductive and capacitive reactance. A generally known formula for the net reactance of this circuit is

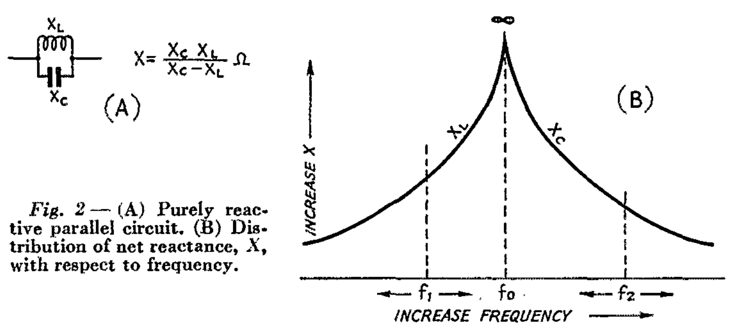

Fig. 2 — **(A) Purely reactive parallel circuit. (B) Distribution of net reactance, *X*, with respect to frequency.**

$$X = \frac{X_C X_L}{X_C - X_L} \text{ ohms.}$$

When $X_L = X_C$, the circuit is parallel resonant, and X equals infinity (theoretically).

Fig. 2B shows the distribution of X at frequencies below and above f_0. At frequencies below f_0, X is inductive, and at frequencies above f_0, X is capacitive. The greater the ohmic values of X_L and X_C, and the less of deviation from f_0, the greater is the magnitude of X.

When this parallel circuit is viewed from two different frequencies, f_1 and f_2, the relations existing between X_{f1} and X_{f2} which are important to these two-band networks are:

1) When f_0 is greater than f_1 and f_2, both X_{f1} and X_{f2} are inductive, but X_{f2} is always greater than KX_{f1}.

2) When f_0 is less than f_1 and f_2, both X_{f1} and X_{f2} are capacitive, but X_{f1} is always greater than KX_{f2}.

3) When f_0 is between f_1 and f_2, X_{f1} is inductive, and X_{f2} is capacitive.

4) When $f_0 = f_1$, X_{f1} is ∞, and X_{f2} is capacitive.

5) When $f_0 = f_2$, X_{f2} is ∞, and X_{f1} is inductive.

Formulas for determining amounts of X_L and X_C required for obtaining desired values of X_{f1} and X_{f2}, consistent with the above five conditions, are easily derived as follows:

At f_1 the net reactance is

$$X_{f1} = \frac{X_C X_L}{X_C - X_L} \text{ ohms.}$$

Since X_L varies directly with frequency and X_C varies inversely with frequency, the net reactance at f_2 is

$$X_{f2} = \frac{\left(\frac{X_C}{K}\right)(KX_L)}{\frac{X_C}{K} - KX_L} \text{ ohms.}$$

Simultaneous solution of these two equations gives the reactance of X_L and X_C at the f_1 frequency as

$$X_L = \frac{X_{f1}(K^2 - 1)}{K\left(K - \frac{X_{f1}}{X_{f2}}\right)} \text{ ohms} \qquad (3)$$

and

$$X_C = \frac{X_{f2}(K^2 - 1)}{\frac{X_{f2}}{X_{f1}} - K} \text{ ohms.} \qquad (4)$$

In the special case of where the desired X_{f1} is ∞ (when $f_0 = f_1$, as in Condition 4), formulas (3) and (4) become equal, and reduce to

$$X_L = X_C = \frac{X_{f2}(1 - K^2)}{K} \text{ ohms.} \qquad (5)$$

In the special case of where the desired X_{f2} is ∞ (when $f_0 = f_2$, as in Condition 5) formulas (3) and (4) reduce to

$$X_L = \frac{X_{f1}(K^2 - 1)}{K^2} \text{ ohms} \qquad (6)$$

and

$$X_C = X_{f1}(K^2 - 1) \text{ ohms,} \qquad (7)$$

respectively.

As in the series circuit of Fig. 1, if the answer to either X_L or X_C turns out negative, it indicates that an unworkable combination of X_{f1} and X_{f2} exists.

Reactive Circuits Containing Resistance

Fig. 3A shows equivalent series and parallel circuits of resistance and inductive reactance, while Fig. 3B shows equivalent series and parallel circuits of resistance and capacitive reactance.

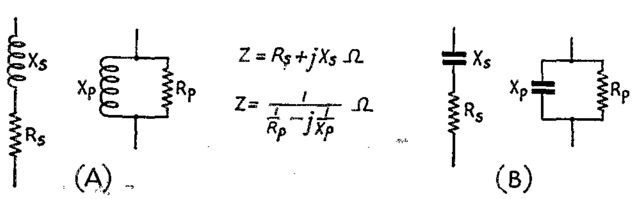

Fig. 3 — Series and parallel equivalents. (A) Resistance and inductive reactance; (B) resistance and capacitive reactance.

Many texts express the impedance of the series circuits as $Z = R + jX_S$ ohms, and that of the parallel circuits as $Z = \frac{1}{\frac{1}{R_P} - j\frac{1}{X_P}}$ ohms.

For each combination of R_S and X_S, there is a combination of R_P and X_P having the same resulting impedance and phase angle. Then when a series circuit and a parallel circuit are electrically equivalent,

$$R_S + jX_S = \frac{1}{\frac{1}{R_P} - j\frac{1}{X_P}}.$$

From this basic expression, it can be shown that the components of the equivalent parallel circuit of a given series circuit are

$$R_P = R_S\left[1 + \left(\frac{X_S}{R_S}\right)^2\right] \text{ ohms} \qquad (8)$$

and

$$X_P = X_S\left[1 + \left(\frac{R_S}{X_S}\right)^2\right] \text{ ohms.} \qquad (9)$$

From that same expression, it can be shown that the components of the equivalent series circuit of a given parallel circuit are

$$R_S = \frac{R_P}{1 + \left(\frac{R_P}{X_P}\right)^2} \text{ ohms} \qquad (10)$$

and

$$X_S = \frac{X_P}{1 + \left(\frac{X_P}{R_P}\right)^2} \text{ ohms.} \qquad (11)$$

Through the use of formulas (8) to (11), inclusive, it is a simple matter to convert a given series circuit to its equivalent parallel circuit, or vice versa.

R.F. Transformer

A careful study of formulas (8) through (11), or a graphical solution of them, will reveal several interesting relations between the four circuit elements in Fig. 3. Those important here are:

1) With a fixed value of R_P, a gradual increase in X_P from zero to infinity, causes a gradual increase in R_S from zero to a maximum, equal to R_P.

2) With a fixed value of R_S, a gradual increase in X_S from zero to infinity, causes a gradual increase in R_P from a minimum, equal to R_S, to a maximum of infinity.

From these two statements, we learn that R_P is always greater than R_S and that the ratio, R_P/R_S, is determined by X_P or X_S.

If R_S and R_P represent values of power-source and load impedance, both of which are purely resistive, the ratio, R_P/R_S, is, in effect, a transformer (impedance) ratio, and when conjugate circuits are used, the impedance of the load is matched to that of the power source. A step-by-step demonstration of this is as follows:

1) The 4-ohm power source in Fig. 4A is shunted by a parallel reactor of such value that the equivalent series circuit contains a resistance component equal to the 2-ohm load. A rearrangement of formula (10), solving for X_P,

$$X_P = R_P \sqrt{\frac{R_S}{R_P - R_S}} \text{ ohms} \qquad (12)$$

gives a value of 4 ohms for the parallel reactor, as shown in Fig. 4B. (Inductive reactance was chosen, arbitrarily.)

2) The equivalent series circuit of the 4-ohm inductive reactance across the 4-ohm resistance is found from formulas (10) and (11), or from formula (11) and a rearrangement of formula (8), solving for X_S,

$$X_S = R_S \sqrt{\frac{R_P}{R_S} - 1} \text{ ohms} \qquad (13)$$

shows the equivalent circuit to be 2 ohms resistance in series with 2 ohms inductive reactance. (Of course, from Step 1 we already knew that the resistance would be 2 ohms.)

3) This equivalent series circuit is substituted for the parallel circuit as in Fig. 4C.

4) A 2-ohm capacitive reactance is placed in series to balance out the 2-ohm inductive reactance as in Fig. 4D.

5) The 2-ohm resistance and 2-ohm inductive reactance in series is replaced by its original parallel equivalent as in Fig. 4E, making a transformer of 2 to 1 impedance ratio, the elements of which are shown in Fig. 4F.

The above results are duplicated, of course, by employing a capacitive shunt reactor and an inductive series reactor. There are other approaches that might be used to demonstrate this transformer ratio, but the one used here fits into the two-band networks nicely.

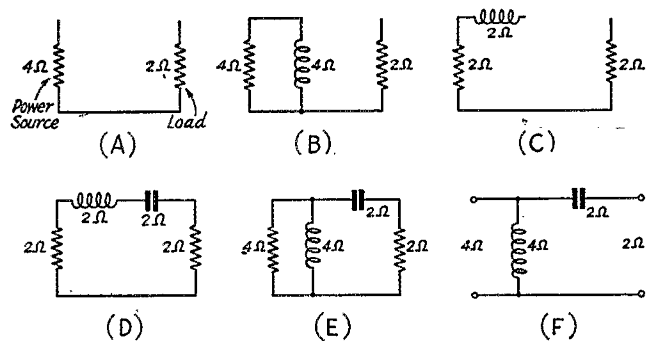

Fig. 4 — Development of an r.f. transformer using shunt and series reactances. See text for discussion.

The type of transformer of Fig. 4F may be used step-up or step-down, the larger impedance always being placed across the parallel reactor, but it must be used only on the frequency at which the reactors are figured; this is strictly a single-frequency transformer. Formulas (12) and (13) give the values of the parallel and series reactors, respectively, for any pair of terminations. Similar formulas were given previously in articles describing certain single-band antenna-matching transformers.[4]

As far as the networks to follow are concerned, it is convenient to view all transformer ratios from the direction that makes them step-down. When doing this, the function of the parallel reactor is to establish the transformer ratio, i.e., the ratio between the resistance connected to the high-impedance side of the transformer and the resistance component of the equivalent series circuit (of the reactor across the resistance connected to the high side); and the function of the series reactor is to balance out the resulting reactance component of that same equivalent series circuit.

Using the above reasoning in the example of Fig. 4, the 4-ohm parallel reactor establishes the transformer ratio, and the 2-ohm series reactor balances out the reactance component of the equivalent series circuit of the 4-ohm inductive reactance in parallel with the 4-ohm power source.

The above analogy serves to separate the functions of the two reactive elements that make up the basic transformer. To understand the "mechanix" of the two-band networks to follow, a substantial knowledge of these transformer effects will be helpful.

Two-Band Networks

The theory of the fundamental circuits of Figs. 1, 2 and 3 combines with the transformer characteristics disclosed in Fig. 4 to make up these two band networks. All of their formulas were derived from basic formulas (1) through (13) of those basic circuits.

There are several possible circuits for two-band antenna networks. However, only three of them are required to cover the combinations of Z_0, Z_1, and Z_2 existing in present-day antennas. Each of these likely combinations will fall into one of the following general cases:

(1) $Z_0 < Z_1$ and Z_2 (2) $Z_1 < Z_0 < Z_2$
(3) Z_1 and $Z_2 < Z_0$

A network capable of handling each of these

[4] Andrew, "An R.F. Matching Network for General Use," *QST*, Oct., 1939; Gadwa, "An Impedance-Matching Transformer," *QST*, Feb., 1943.

three general cases will be described, separately, in turn.

For the sake of example, a few common radiating systems will be classified into the three general cases. Reference to a long-wire system includes the "V" and the unterminated rhombic as well as the simple long wire, which will be considered as a wire one wave-length or more long at f_1. Factors such as proximity to ground and other objects affect the value of d.p.i., and under extreme conditions a certain radiating system might not fall into the general case under which it is grouped. This will be disclosed, however, when determining the values of d.p.i., which will be discussed later.

Case of $Z_0 < Z_1$ and Z_2

This general case covers such systems as:

1) A current-fed half-wave doublet at f_1, also operating on any harmonic, f_2, of f_1, using 53-ohm line. When $K = 2$, this system is two half waves in phase at f_2, voltage fed.

2) A center-fed two half-waves-in-phase (collinear) system at f_1, also operating on any harmonic, f_2, of f_1, using any type of line. This antenna is voltage fed at both f_1 and f_2.

3) A long-wire system, current fed at f_1, also operating on any harmonic, f_2, of f_1, using 53- or 75-ohm line.

4) A long-wire system, voltage fed at f_1, also operating on any harmonic, f_2, of f_1, using any type of line. This antenna is voltage fed at f_2, also.

Fig. 5 shows a suitable network for this general case of $Z_0 < Z_1$ and Z_2.

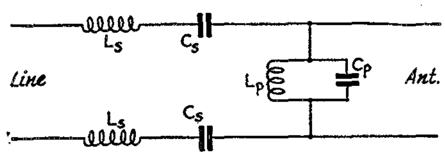

Fig. 5 — Two-band network suitable when the line impedance is less than the driving-point impedances.

The reactance of L_P and C_P in Fig. 5 is proportioned so that:

1) At f_1, the net reactance, X_{P1}, is inductive and of such magnitude that the resistance component of the equivalent series circuit of X_{P1} in parallel with Z_1 equals Z_0; and

2) At f_2, the net reactance, X_{P2}, is capacitive and of such magnitude that likewise establishes the correct transformer ratio between Z_2 and Z_0.

Since X_{P1} is inductive and X_{P2} is capacitive, condition (3) of the basic parallel circuit of Fig. 2 is satisfied.

From basic formula (12), the required net reactance of L_P and C_P at f_1 is

$$X_{P1} = Z_1 \sqrt{\frac{Z_0}{Z_1 - Z_0}} \text{ ohms;}$$

and at f_2 it is

$$X_{P2} = -Z_2 \sqrt{\frac{Z_0}{Z_2 - Z_0}} \text{ ohms.}$$

Then, from basic formulas (3) and (4), the reactance of L_P and C_P at the f_1 frequency is

$$X_{LP} = \frac{X_{P1}(K^2 - 1)}{K\left(K - \frac{X_{P1}}{X_{P2}}\right)} \text{ ohms} \quad (14)$$

and

$$X_{CP} = \frac{X_{P2}(K^2 - 1)}{\frac{X_{P2}}{X_{P1}} - K} \text{ ohms,} \quad (15)$$

respectively.

The reactances of both sets of L_S and C_S are simultaneously proportioned so that:

1) At f_1, the total net reactance, X_{B1}, is equal in magnitude but opposite in sign to the reactance component of the equivalent series circuit of X_{P1} in parallel with Z_1; and

2) At f_2, the total net reactance, X_{B2}, likewise balances out the reactance component of the equivalent series circuit of X_{P2} in parallel with Z_2.

Then, X_{B1} must be capacitive, and X_{B2} inductive. This satisfies condition (3) of the basic series circuit of Fig. 1. From basic formula (13) the required total net reactance of L_S and C_S at f_1 is

$$X_{B1} = -Z_0 \sqrt{\frac{Z_1}{Z_0} - 1} \text{ ohms,}$$

and at f_2 it is

$$X_{B2} = Z_0 \sqrt{\frac{Z_2}{Z_0} - 1} \text{ ohms.}$$

Then, from basic formulas (1) and (2), the reactance of each L_S and each C_S at the f_1 frequency is

$$X_{LS} = \frac{KX_{B2} - X_{B1}}{2(K^2 - 1)} \text{ ohms} \quad (16)$$

and

$$X_{CS} = \frac{K(X_{B2} - KX_{B1})}{2(K^2 - 1)} \text{ ohms,} \quad (17)$$

respectively.

[*EDITOR'S NOTE — The second part of this article, to appear in a subsequent issue, will cover the two remaining cases, Z_1 and $Z_2 < Z_0$, and $Z_1 < Z_0 < Z_2$, and will discuss practical adjustment problems.*]